

Influence of polarization mode competition on the synchronization of two unidirectionally coupled vertical-cavity surface-emitting lasers

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We analyze theoretically the effect of polarization mode competition on the synchronization of two unidirectionally coupled vertical-cavity surface-emitting lasers (VCSELs). Chaos in the master laser is induced by delayed optical feedback, and the slave laser is subject to isotropic optical injection from the master VCSEL. We show that the synchronization quality can be enhanced when the chaotic regime in the master VCSEL involves both fundamental orthogonal linearly polarized modes. © 2007 Optical Society of America
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Synchronization of chaotic systems has been a subject of both theoretical and experimental studies [1]. In particular, the synchronization between two chaotic semiconductor lasers has attracted considerable attention owing to its application in secure chaos-based optical communications [2]. Among the diode lasers, vertical-cavity surface-emitting lasers (VCSELs) exhibit several advantages such as a low threshold current, circular, weakly diverging output beam, and easy production of two-dimensional arrays. Moreover, VCSELs may exhibit intriguing switching between two orthogonal linearly polarized (LP) modes [3,4]. In spite of these interesting properties, studies on chaos synchronization in coupled VCSELs remain scarce. Recent theoretical works have focused on the synchronization properties in polarization-dependent optical injection schemes [5] or using multitransverse but single LP mode VCSELs [6]. Chaos synchronization has also been very recently shown experimentally [7]. The growing interest in implementing chaos communication using VCSELs motivates further investigations to better understand the influence of polarization dynamics on the synchronization properties.

In this Letter, we present a theoretical investigation of the effect of polarization mode competition on the synchronization characteristics of two unidirectionally coupled VCSELs. The master VCSEL only is rendered chaotic by optical feedback, and its chaotic output is coupled to the slave VCSEL by use of an isotropic optical injection. We show that, depending on the injection conditions, the synchronization quality can be strongly enhanced when the master laser and therefore also the synchronized slave exhibit two LP mode dynamics.

Our rate equation model extends the standard spin flip model for single transverse mode VCSELs [8]:

$$\begin{aligned} \dot{E}_{x,y}^m &= \kappa(1+i\alpha)[(N^m-1)E_{x,y}^m \pm in^m E_{y,x}^m] \\ &\mp (\gamma_a + i\gamma_p)E_{x,y}^m + fE_{x,y}^m(t-\tau)e^{i\omega_m\tau} + F_{x,y}^m, \quad (1) \end{aligned}$$

$$\begin{aligned} \dot{E}_{x,y}^s &= \kappa(1+i\alpha)[(N^s-1)E_{x,y}^s \pm in^s E_{y,x}^s] \\ &\mp (\gamma_a + i\gamma_p)E_{x,y}^s - i\Delta\omega E_{x,y}^s + \eta E_{x,y}^m(t) + F_{x,y}^s, \quad (2) \end{aligned}$$

$$\begin{aligned} \dot{N}^{m,s} &= -\gamma[N^{m,s} - \mu + N^{m,s}(|E_x^{m,s}|^2 + |E_y^{m,s}|^2)] \\ &- i\gamma n^{m,s}(E_y^{m,s} E_x^{m,s*} - E_x^{m,s} E_y^{m,s*}), \quad (3) \end{aligned}$$

$$\begin{aligned} \dot{n}^{m,s} &= -\gamma_s n_s^{m,s} - \gamma[n^{m,s}(|E_x^{m,s}|^2 + |E_y^{m,s}|^2)] \\ &- i\gamma N^{m,s}(E_y^{m,s} E_x^{m,s*} - E_x^{m,s} E_y^{m,s*}). \quad (4) \end{aligned}$$

The superscripts m and s are used for the master and slave VCSELs, respectively. $E_x^{m,s}$ and $E_y^{m,s}$ are the slowly varying amplitudes of the x - and y -LP field components. $N^{m,s}$ is the total carrier inversion between conduction and valence bands, while $n^{m,s}$ accounts for the difference between carrier inversions with opposite spins. κ is the photon decay rate, γ is the carrier decay rate, and γ_s accounts for microscopic processes leading to the homogenization of carrier spin. α is the linewidth enhancement factor, and μ is the normalized injection current ($\mu=1$ at threshold). γ_a and γ_p model the linear cavity dichroism and phase anisotropy, respectively. $\Delta\omega = \omega^m - \omega^s$ is the frequency detuning. f is the feedback rate, and η is the injection rate. τ is the delay time in the external cavity. We consider a zero flight time between master and slave lasers. Spontaneous emission noise is modeled by Langevin sources: $F_x^{m,s} = \sqrt{\beta_{sp}/2}(\sqrt{N^{m,s} + n^{m,s}}\xi_1^{m,s} + \sqrt{N^{m,s} - n^{m,s}}\xi_2^{m,s})$ and $F_y^{m,s} = -i\sqrt{\beta_{sp}/2}(\sqrt{N^{m,s} + n^{m,s}}\xi_1^{m,s} - \sqrt{N^{m,s} - n^{m,s}}\xi_2^{m,s})$, with $\xi_1^n, \xi_1^s, \xi_2^n, \xi_2^s$ independent Gaussian white noise with zero mean and unitary variance [8]. We keep the following parameters fixed: $\alpha=3$, $\gamma=1$ ns⁻¹, $\gamma_s=50$ ns⁻¹, $\gamma_a=0.1$ ns⁻¹, $\kappa=300$ ns⁻¹, $\Delta\omega=0$, $\tau=3$ ns, $\omega_m\tau=6$ rad, $\mu=1.2$.

In our isotropic optical feedback/injection configuration, the two VCSELs can exhibit either an anticipating or isochronous (injection-locking) type of synchronization [9]. Anticipating (perfect) synchronization is achieved when $I_{x,y}^m(t) = I_{x,y}^s(t - \tau)$, $N^m(t) = N^s(t - \tau)$, and $n^m(t) = n^s(t - \tau)$, with $I_{x,y}^{m,s} = |E_{x,y}^{m,s}|^2$. Necessary conditions for the existence of such a solution are that both lasers exhibit the same device parameters, bias currents, zero detuning, and also $f = \eta$. The isochronous synchronization solution is of the type $I_{x,y}^m(t) = a I_{x,y}^s(t)$, with a being a constant. To discriminate between isochronous and anticipative synchronizations, and to evaluate the synchronization quality, we use the following correlation coefficients, respectively:

$$C_{1x,y} = \frac{\langle [I_{x,y}^m(t) - \langle I_{x,y}^m \rangle][I_{x,y}^s(t) - \langle I_{x,y}^s \rangle] \rangle}{\{ \langle [I_{x,y}^m(t) - \langle I_{x,y}^m \rangle]^2 \rangle \langle [I_{x,y}^s(t) - \langle I_{x,y}^s \rangle]^2 \rangle \}^{1/2}}, \quad (5)$$

$$C_{2x,y} = \frac{\langle [I_{x,y}^m(t + \tau) - \langle I_{x,y}^m \rangle][I_{x,y}^s(t) - \langle I_{x,y}^s \rangle] \rangle}{\{ \langle [I_{x,y}^m(t) - \langle I_{x,y}^m \rangle]^2 \rangle \langle [I_{x,y}^s(t) - \langle I_{x,y}^s \rangle]^2 \rangle \}^{1/2}}. \quad (6)$$

To examine the effect of the polarization mode competition on the synchronization quality we consider first $\gamma_p = 6 \text{ ns}^{-1}$ such that the free-running VCSELs operate in a parameter region where both x - and y -LP modes are stable [8]. We first analyze mode competition without spontaneous emission noise (i.e., $\beta_{sp} = 0$). With these parameters, we find that the dynamics induced by the delayed optical feedback in the master laser will also exhibit bistability. Depending on the system's initial conditions, the master laser can exhibit either a chaotic dynamics involving both LP modes or a single-mode chaotic dynamics, as shown in Figs. 1(a1) and 1(a3), and Figs. 1(b1) and 1(b3), respectively. We then analyze in each case the slave laser synchronization as we progressively increase the injection rate. We consider the injection and feedback conditions such that the necessary conditions for anticipative synchronization are met. Tak-

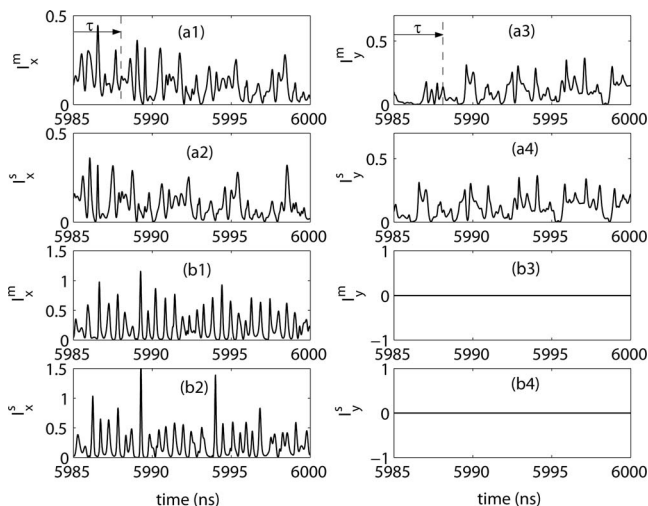


Fig. 1. Time traces of master and slave laser LP mode intensities for $\eta = f = 6 \text{ ns}^{-1}$: (a1)–(a4) chaos in both x - and y -LP modes and (b1)–(b4) single-mode chaos.

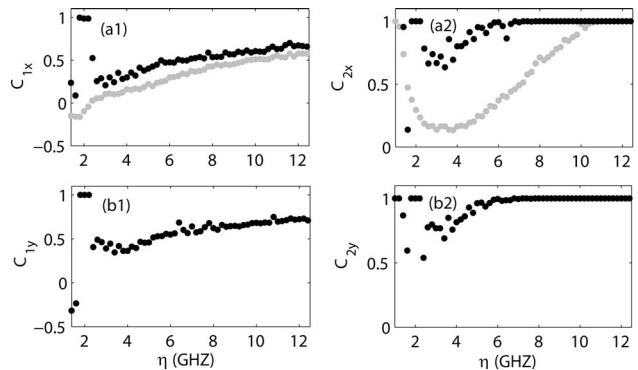


Fig. 2. Evolution of the correlation coefficients related to isochronous (a1) and (b1) and anticipative (a2) and (b2) synchronization as a function of η . The gray (black) color corresponds to the case of single-mode (two-mode) chaos.

ing for example $\eta = f = 6 \text{ ns}^{-1}$, we observe first in Figs. 1(a2) and 1(a4) that the slave laser exhibits an almost perfect anticipative synchronization ($C_{2x} = 0.94$) for each pair of corresponding LP modes. However, we find interestingly that for the same parameters but considering a single-mode chaotic dynamics in the master laser leads to a significant decrease of the synchronization quality ($C_{2x} = 0.36$); see Figs. 1(b1) and 1(b2). The master laser total intensity exhibits more frequent power dropouts in the one-mode case than in the two-mode case, where, as a result of antiphase mode competition, the decrease of, e.g., x -LP mode intensity is accompanied by a buildup of y -LP mode intensity. However, we find that the desynchronization events are not necessarily related to these power dropouts in the injected total intensity. Therefore the change of synchronization quality between the cases of Figs. 1(a1)–1(a4) and 1(b1)–1(b4) is attributed not to a change of injection level but rather to the intrinsic polarization competition mechanism.

Figure 2 shows the evolution of the isochronous and the anticipative correlation coefficients when the injection rate η is varied but still equal to the feedback rate f . In Figs. 2(a1) and 2(a2), we add gray curves that show the evolution of C_{1x} and C_{2x} in the case of single x -LP mode chaotic dynamics. In the whole range of η , $C_2 > C_1$, whatever the LP modes that are analyzed (x or y) and independently of the single x -LP mode or two LP mode cases in the laser dynamics (black or gray curves). The lasers therefore exhibit anticipative synchronization. The still large isochronous correlation coefficient C_1 can be related to the presence of a periodicity at τ in the intensity time traces. For $\eta < 3 \text{ ns}^{-1}$ the lasers exhibit either weakly synchronized irregular dynamics or locked steady states (see the small range of η with perfect synchronization in the two LP mode case). For larger η the synchronization quality improves with η . Indeed, the lasers exhibit desynchronization bursts, but the average time between them increases as η increases, leading to better synchronization over a given time window. The comparison between the black and gray curves in Figs. 2(a1) and 2(a2) unveils, moreover, that the correlation coefficients (and

therefore the synchronization quality) improve in the two LP mode case, confirming the observation of Fig. 1 in the whole range of η .

If we now include spontaneous emission noise, we find that the master laser exhibits only a two-mode chaotic dynamics, independently of the system initial condition. Figure 3 plots C_{1x} and C_{2x} as a function of η and for the same parameters as in Fig. 2 but $\beta_{sp} = 10^{-6} \text{ ns}^{-1}$. By comparing Fig. 3 with Fig. 2, we find that the synchronization quality is slightly degraded by the inclusion of the spontaneous emission noise, as also seen in other systems [10]. However, both correlation coefficients are still larger than those obtained in the case of a deterministic single-mode chaotic dynamics [compare Figs. 3(a1) and 3(a2) with the gray curves in Figs. 2(a1) and 2(a2), respectively]. The enhancement of synchronization quality, in the case of a two-mode dynamics, can therefore not be attributed to noise, which typically has the opposite effect, but rather to the two-mode dynamics itself.

As a final test, we investigate how the anticipative synchronization quality evolves when the system is driven from its bistability region to a region where only a single-mode solution exists even in the presence of spontaneous emission. In fact, we show that, by varying γ_p , a robust feedback-induced single-mode chaotic dynamics can be obtained. Figure 4 shows the evolution of C_{2x} when γ_p is changed but for a fixed injection parameter ($\eta=f=8 \text{ ns}^{-1}$). We start with $\gamma_p = 6 \text{ rad ns}^{-1}$, for which the system operates in a bistability region. As discussed in Fig. 3, the system exhibits a chaotic dynamics in both LP modes, and a very good anticipative synchronization is achieved. If γ_p is then increased, we observe an abrupt degradation of the synchronization around $\gamma_p = 10 \text{ rad ns}^{-1}$. In fact, for $\gamma_p > 10 \text{ rad ns}^{-1}$ only the x -LP mode is stable in the free-running VCSEL [8]. Then the system exhibits a robust single-mode dynamics even with noise, and the decrease of synchronization quality corresponds to this transition from two-mode to one-mode dynamics. As γ_p is increased further, the synchronization quality does not vary anymore.

Our conclusions remain valid also when accounting for frequency detuning and additional optical feedback on the slave laser (closed loop). However, as already observed for edge-emitting lasers [11], an increase of the detuning and/or an increase of the slave laser feedback rate leads to a decrease of the synchronization quality, for both one-mode and two-mode dynamics cases.

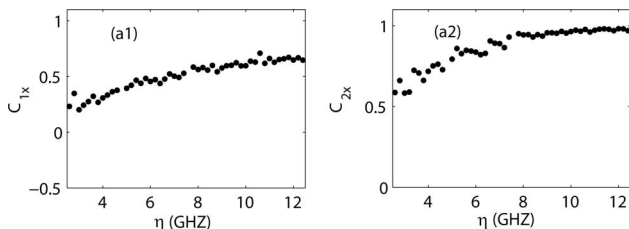


Fig. 3. Same as Fig. 2 but with $\beta_{sp} = 10^{-6} \text{ ns}^{-1}$.

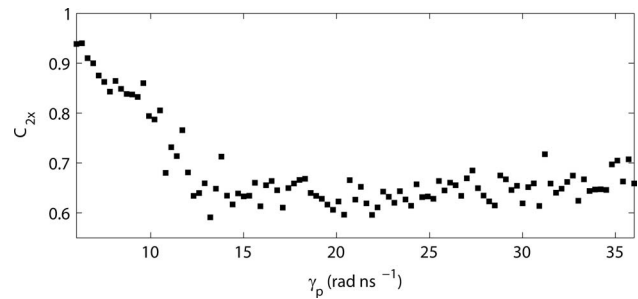


Fig. 4. Evolution of C_{2x} when increasing γ_p .

In conclusion, we have shown that the synchronization quality between unidirectionally coupled VCSELs can be significantly enhanced when the feedback-induced chaos in the master laser involves both orthogonal LP fundamental transverse modes. This effect is particularly clear when tuning the VCSEL birefringence and even in the realistic presence of spontaneous emission noise. Our conclusions motivate dedicated experiments where the linear cavity anisotropies, and therefore the VCSEL polarization mode behavior, can be modified by strain and/or temperature effects [12]. Our results are also thought to be important in the context of emerging VCSEL chaos-based communication systems.

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References

1. L. M. Pecora and T. L. Carroll, *Phys. Rev. Lett.* **64**, 821 (1990).
2. C. R. Mirasso, P. Colet, and P. Garcia-Fernandez, *IEEE Photon. Technol. Lett.* **8**, 299 (1996).
3. K. D. Choquette, R. P. Schneider, L. K. Lear, and R. E. Leibenguth, *IEEE J. Sel. Top. Quantum Electron.* **1**, 661 (1995).
4. M. Sciamanna, K. Panajotov, H. Thienpont, I. Veretennicoff, P. Mégret, and M. Blondel, *Opt. Lett.* **28**, 1543 (2003).
5. R. Ju, P. Spencer, and K. A. Shore, *IEEE J. Quantum Electron.* **41**, 1461 (2005).
6. M. S. Torre, C. Masoller, and K. A. Shore, *J. Opt. Soc. Am. B* **21**, 1772 (2004).
7. Y. Hong, M. W. Lee, P. Spencer, and K. A. Shore, *Opt. Lett.* **29**, 1215 (2004).
8. J. M. Regalado, F. Prati, M. San Miguel, and N. B. Abraham, *IEEE J. Quantum Electron.* **33**, 765 (1997).
9. A. Locquet, F. Rogister, M. Sciamanna, P. Mégret, and M. Blondel, *Phys. Rev. E* **64**, 045203(R) (2001).
10. Y. Liu, H. F. Chen, J. M. Liu, P. Davis, and T. Aida, *Phys. Rev. A* **63**, 031802 (2001).
11. R. Vicente, T. Perez, and C. R. Mirasso, *IEEE J. Quantum Electron.* **38**, 1197 (2002).
12. K. Panajotov, B. Nagler, G. Verschaffelt, A. Georgievski, H. Thienpont, J. Danckaert, and I. Veretennicoff, *Appl. Phys. Lett.* **77**, 1590 (2000).